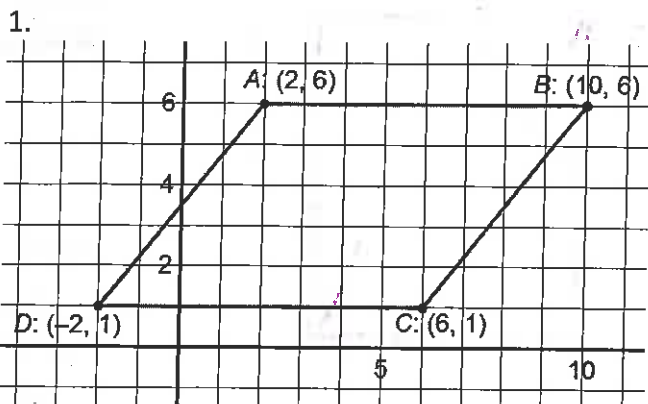


Parallelograms

Parallelograms:

A **Parallelogram** is a *quadrilateral* (4 sided shape) with 2 pairs of parallel sides.



a. To prove quadrilateral ABCD is a parallelogram, you need to first show $\overline{AB} \parallel \overline{DC}$. Based on the picture, how do you know that $\overline{AB} \parallel \overline{DC}$? Explain your reasoning.

\overline{AB} and \overline{DC} are horizontal lines and horizontal lines are always parallel.

b. You will also need to show $\overline{DA} \parallel \overline{CB}$. This can't be assumed from the picture. Why? What will you need to show as evidence that $\overline{DA} \parallel \overline{CB}$? Explain your reasoning.

$\overline{DA} \parallel \overline{CB}$ can't be assumed because they are not vertical or horizontal lines. They may or may not be parallel.

I can show that their slopes are equal as evidence to show they are parallel.

c. Prove quadrilateral ABCD is a parallelogram.

$\overline{AB} \parallel \overline{DC}$ because they are both horizontal.

$$\left. \begin{array}{l} \text{Slope } \overline{AD} = \frac{6-1}{2-(-2)} = \frac{5}{4} \\ \text{Slope } \overline{CB} = \frac{6-1}{10-6} = \frac{5}{4} \end{array} \right\} \overline{AD} \parallel \overline{CB} \text{ because they have equal slopes.}$$

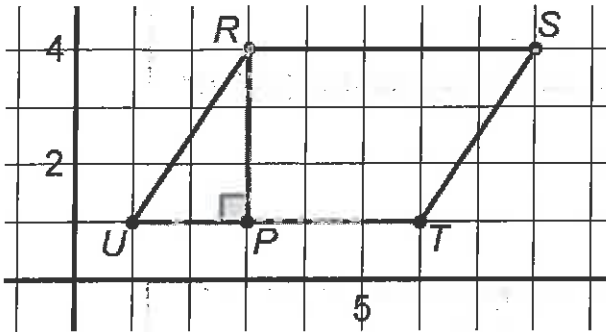
Quad. ABCD is a parallelogram because it has 2 pairs of parallel sides.

Parallelogram Perimeter and Area:

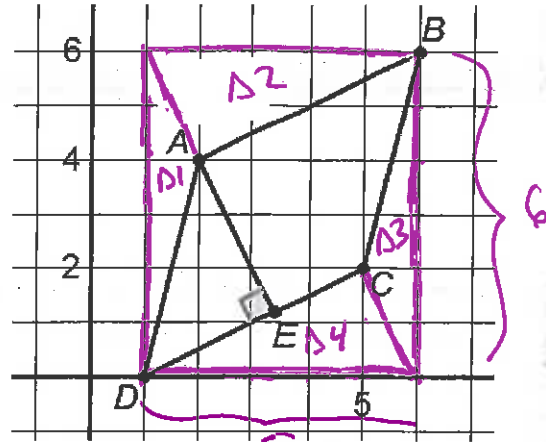
The area of a parallelogram: $A = bh$

where b is the base and h is the height.

The **height** (or *Altitude*) of a parallelogram is a segment drawn from a vertex of the parallelogram perpendicular to a point on the side opposite the vertex. Altitudes \overline{RP} and \overline{AE} have been drawn in each of the parallelograms pictured below.



Picture 1



Picture 2

2. Find the area of parallelograms RSTU and ABCD.

height $RP = 3$
base $UT = 5$

Area //ogram RSTU
 $= (3)(5)$
 $= 15 \text{ sq. units.}$

height AE is not easily found because I don't know the point E .

I'll use the box method instead.

Area box $= (5)(6) = 30$

Area $\Delta 1 = \frac{1}{2}(6 \times 1) = 3$

$\Delta 2 = \frac{1}{2}(5 \times 2) = 5$

$\Delta 3 = \frac{1}{2}(6 \times 1) = 3$

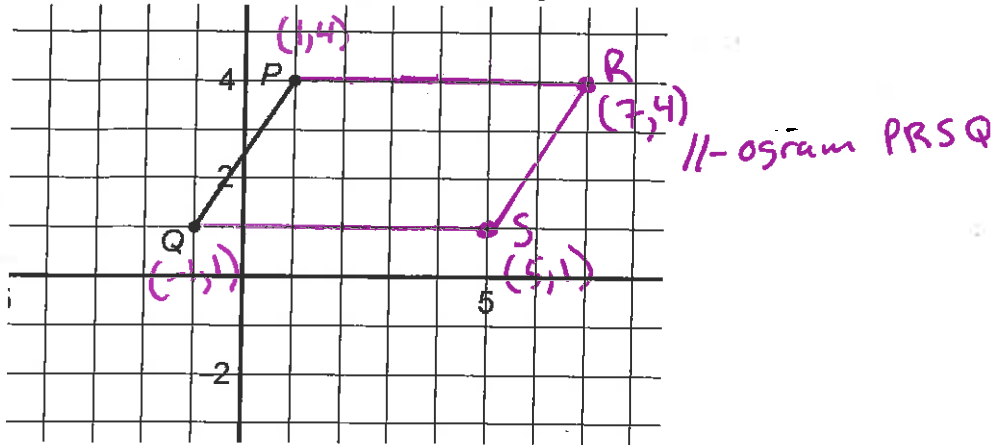
$\Delta 4 = \frac{1}{2}(5 \times 2) = 5$

Area //ogram ABCD $= 30 - (3+5+3+5)$
 $= 14 \text{ sq. units.}$

Other Parallelogram Properties:

Parallelograms have a variety of other useful properties. Let's look at two of them.

3. On the grid, draw any parallelogram using \overline{PQ} as one of its sides.



4. How do you know your quadrilateral is a parallelogram? Explain your reasoning.

I know $\overline{PR} \parallel \overline{QS}$ because they are both horizontal lines.

I know $\overline{PQ} \parallel \overline{RS}$ because I used the slope of $\overline{PQ} = \frac{3}{2}$

to make \overline{RS} have the same slope.

5. "The opposite sides of a parallelogram are congruent."

Prove this parallelogram property is true using your parallelogram.

$$\left. \begin{array}{l} PR = 6 \\ QS = 6 \end{array} \right\} \overline{PR} \cong \overline{QS}$$

$$PQ = \sqrt{(1 - (-1))^2 + (4 - 1)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$RS = \sqrt{(7 - 5)^2 + (4 - 1)^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\text{So, } \overline{PQ} \cong \overline{RS}$$

the opposite sides of ||-ogram PQRS are \cong because they are the same length.

4. "The diagonals of a parallelogram bisect each other."

Prove this property is true using your parallelogram. (Hint: To bisect a segment means to cut through its midpoint.)

$$\text{midpoint of Diagonal } \overline{PS} = \left(\frac{1+5}{2}, \frac{4+1}{2} \right) = (3, 2.5)$$

$$\text{midpoint of Diagonal } \overline{RQ} = \left(\frac{7+1}{2}, \frac{4+1}{2} \right) = (3, 2.5)$$

Since \overline{PS} and \overline{RQ} have the same midpoint, that means that they bisect each other.

